M251 Practice Exam 3 sections 7.4, 8.1-8.3

Name: Key Date: ____________

1. Find the indefinite integral.

$\int 8(x-5)^7\,dx$

$u = x-5$
$du = dx$

$\int 8u^7 du = \frac{8u^8}{8} + c = u^8 + c = (x-5)^8 + c$

2. Find the indefinite integral.

$\int \frac{2}{(r-1)^4}\,dr$

$u = r-1$
$du = dr$

$\int \frac{2}{u^4} du = \int 2u^{-4} du = \frac{2u^{-3}}{-3} + c = -\frac{2}{3(r-1)^3} + c$

3. Find the indefinite integral.

$\int \left[r - \frac{2}{(r-5)^8}\right] dr = \int r\,dr - 2\int \frac{dr}{(r-5)^8}$

$u = r-5$
$du = dr$

$\frac{r^2}{2} - 2\int u^{-8} du = \frac{r^2}{2} - \frac{2u^{-7}}{-7} + c = \frac{r^2}{2} + \frac{2}{7(r-5)^7} + c$

4. Find the indefinite integral.

$\int \frac{z^2}{z+7}\,dz$

$u = z+7$
$du = dz$
$u-7 = z$

$\int \frac{(u-7)^2}{u} du = \int \frac{u^2 - 14u + 49}{u} du$

$= \int u - 14 + \frac{49}{u} du = \frac{u^2}{2} - 14u + 49\ln|u|$

$= \frac{(z+7)^2}{2} - 14z + 49\ln|z+7| + c$

$= \frac{(z-7)^2}{2} + 49\ln|z+7| + c$

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5. Find the indefinite integral.

$\int \frac{6y}{y+3}\,dy$

$u = y+3$
$du = dy$
$u-3 = y$

$\int \frac{6(u-3)}{u} du = \int 6 - \frac{18}{u} du$

$= 6u - 18\ln|u| + c$

$= 6y - 18\ln|y+3| + c$

6. **Find the indefinite integral.**

$$\int \frac{\cot(12/w^2)}{w^3}\,dw$$

$u = \frac{12}{w^2}$

$du = \frac{-24}{w^3}dw$

$\frac{du}{-24} = \frac{dw}{w^3}$

$-\frac{1}{24}\int \cot(u)\,du$

$\frac{-\ln|\sin u|}{24} + c = \frac{-1}{24}\ln\left|\sin\left(\frac{12}{w^2}\right)\right| + c$

7. **Find the definite integral.**

$$\int_0^{15} \frac{4x}{\sqrt{x^2+64}}\,dx$$

$u = x^2+64$

$du = 2x\,dx$

$\int_{x=0}^{x=15} \frac{2du}{\sqrt{u}} = \left[2\frac{u^{\frac{1}{2}}}{\frac{1}{2}}\right]_{x=0,\ u=64}^{x=15,\ u=289} = 4\left[\sqrt{289} + \sqrt{64}\right]$

$= 4(17-8) = 36$

8. **Find the definite integral.**

$$\int_0^2 x^3 e^{-x^4}\,dx$$

$u = -x^4$

$du = -4x^3dx$

$-\frac{1}{4}\int_{x=0}^{x=2} e^u du = \left[-\frac{1}{4}e^{-x^4}\right]_{x=0}^{x=2} = -\frac{1}{4}\left(e^{-16} - e^0\right) = \frac{1}{4} - \frac{1}{4e^{16}}$

9. **Find the indefinite integral.**

$$\int x^3 e^{6x^2}\,dx$$

$w = 6x^2$

$dw = 12x\,dx$

$\frac{dw}{12} = x\,dx \qquad \frac{w}{6} = x^2$

$\int \frac{w}{6}e^w\frac{dw}{12} = \frac{1}{6\cdot 12}\left[we^w - \int e^w dw\right]$

$u = w \quad dv = e^w dw$

$du = dw \quad v = e^w$

$\frac{1}{6\cdot 12}(w-1)e^w$

$\boxed{= \frac{1}{72}(6x^2-1)e^{6x^2} + c}$

10. **Find the indefinite integral.**

$$\int \frac{8x^2}{e^x}\,dx$$

$u = 8x^2 \quad dv = e^{-x}dx$

$du = 16x\,dx \quad v = -e^{-x}$

$-8x^2e^{-x} + 16\int xe^{-x}dx$

$\hat{u} = x \quad d\hat{v} = e^{-x}dx$

$d\hat{u} = dx \quad \hat{v} = -e^{-x}$

$-8x^2e^{-x} + 16\left[-xe^{-x} - \int -e^{-x}dx\right]$

$-8x^2e^{-x} - 16xe^{-x} - 16e^{-x} + c$

$\frac{-8}{e^x}(x^2+2x+2) + c$

11. **Find the indefinite integral.**

$$\int x^3 \ln x\,dx$$

$u = \ln x \quad dv = x^3dx$

$du = \frac{dx}{x} \quad v = \frac{x^4}{4}$

$\frac{x^4}{4}\ln(x) - \int \frac{x^4}{4}\frac{dx}{x} = \frac{x^4}{4}\ln(x) - \frac{x^4}{16}$

$= \frac{x^4}{16}(4\ln(x) - 1) + c$

12. Find the indefinite integral.
$\int \frac{3(\ln x)^2}{x^2}dx$

$u=(\ln x)^2$ $dv=3x^{-2}dx$
$du=2\frac{(\ln x)}{x}dx$ $v=-3x^{-1}$

$\frac{-3(\ln x)^2}{x} - \int \frac{-6\ln x}{x^2}dx$

$\hat{u}=\ln x$ $dv=x^{-2}dx$
$d\hat{u}=\frac{dx}{x}$ $v=-x^{-1}$

$= \frac{-3(\ln x)^2}{x}+6\left[-\frac{\ln x}{x}-\int\frac{-dx}{x^2}\right]$

$= -\frac{3(\ln x)^2}{x}-\frac{6\ln x}{x}-\frac{6}{x}+c$

13. Find the indefinite integral.
$\int u\ln(u+6)\,du$

$x=u+6$
$dx=du$
$\int(x-6)\ln x\,dx$

$\hat{u}=\ln x$ $dv=(x-6)dx$
$d\hat{u}=\frac{dx}{x}$ $v=\frac{x^2}{2}-6x$

$\left(\frac{x^2}{2}-6x\right)\ln x-\int\left(\frac{x^2}{2}-6x\right)\frac{dx}{x}$

$\left(\frac{x^2}{2}-6x\right)\ln x+\int 6-\frac{x}{2}\,dx$

$\left(\frac{x^2}{2}-6x\right)\ln x+6x-\frac{x^2}{4}+c$

$=\left[\frac{(u+6)^2}{2}-6(u+6)\right]\ln(u+6)+6u-\frac{(u+6)^2}{4}+c$

$\frac{u^2-36}{2}\ln(u+6)+6u-\frac{(u+6)^2}{4}+c$

14. Find the indefinite integral.
$\int\frac{\ln q}{2q^3}dq$

$u=\ln q$ $dv=\frac{q^{-3}}{2}dq$
$du=\frac{dq}{q}$ $v=-\frac{q^{-2}}{4}$

$\frac{-\ln q}{4q^2}+\int\frac{dq}{4q^3}=\frac{-\ln q}{4q^2}-\frac{1}{8q^2}+c=\frac{-1}{8q^2}(2\ln q+1)+c$

15. Find the indefinite integral.
$\int v\sqrt{2v-9}\,dv$

$u=2v-9$ $v=\frac{u+9}{2}$
$du=2dv$
$\frac{du}{2}=dv$

$\frac{1}{2}\int(u+9)\sqrt{u}\frac{du}{2}=\frac{1}{4}\int u^{\frac{3}{2}}+9u^{\frac{1}{2}}du$

$\frac{1}{4}\left[\frac{u^{5/2}}{5/2}+\frac{9u^{3/2}}{3/2}\right]=\frac{u^{3/2}}{2}\left[\frac{u}{5}+3\right]+c=\frac{(2v-9)^{3/2}}{2}\left(\frac{2v-9}{5}+3\right)$

$=\frac{(2v-9)^{3/2}(v+3)}{5}+c$

16. Find the indefinite integral.
$\int\frac{t}{\sqrt{1+3t}}dt$

$u=1+3t$
$du=3dt$
$\frac{u-1}{3}=t$

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$\int\frac{u-1}{3}u^{-\frac{1}{2}}\frac{du}{3}=\frac{1}{9}\int u^{\frac{1}{2}}-u^{-\frac{1}{2}}du$

$=\frac{1}{9}\left(\frac{u^{3/2}}{3/2}-\frac{u^{1/2}}{1/2}\right)=\frac{2}{27}\sqrt{u}(u-3)+c$

$=\frac{2}{27}\sqrt{1+3t}\,(3t-2)+c$

17. Find the indefinite integral.
$f(x)=\int e^{-7n}\sin 5n\,dn$

$u=\sin 5n$ $dv=e^{-7n}dn$
$du=5\cos 5n\,dn$ $v=\frac{e^{-7n}}{-7}$

$-\frac{(\sin 5n)e^{-7n}}{7}+\frac{5}{7}\int e^{-7n}\cos 5n\,dn$

$\int e^{-7n}\cos 5n\,dn$
$u=\cos 5n$ $dv=e^{-7n}dn$
$du=-5\sin 5n$ $v=\frac{e^{-7n}}{-7}$

$-\frac{\cos 5n}{7}e^{-7n}-\frac{5}{7}\int e^{-7n}\sin 5n\,dn$

$f(x)=-\frac{\sin(5n)}{7}e^{-7n}-\frac{5\cos(5n)}{7^2}e^{-7n}-\frac{25}{49}f(x)$

$\frac{74}{49}f(x)=\left(-\frac{\sin(5n)}{7}-\frac{5\cos 5n}{7^2}\right)e^{-7n}$

$f(x)=\frac{-7\sin(5n)-5\cos(5n)}{74}e^{-7n}+c$

18. Find the indefinite integral.

$f(x) = \int e^{9u}\cos 6u\, du$

$w = e^{9u}$ $dv = \cos 6u\,du$

$dw = 9e^{9u}du$ $v = \frac{\sin 6u}{6}$

$\frac{1}{6}e^{9u}\sin 6u - \frac{9}{6}\int e^{9u}\sin 6u\,du$

$\int e^{9u}\sin 6u\,du$

$w = e^{9u}$ $dv = \sin 6u\,du$

$dw = 9e^{9u}du$ $v = -\frac{\cos 6u}{6}$

$-\frac{e^{9u}\cos 6u}{6} + \frac{9}{6}\int e^{9u}\cos 6u\,du$

$f(x) = \left(\frac{1}{6}\sin 6u + \frac{9}{36}\cos 6u\right)e^{9u} - \left(\frac{9}{6}\right)^2 f(x)$

$(36+81)f(x) = (6\sin 6u + 9\cos 6u)e^{9u}$

$f(x) = \frac{e^{9u}(6\sin 6u + 9\cos 6u)}{9 \cdot 13}$

$= \frac{e^{9u}(2\sin 6u + 3\cos 6u)}{39} + C$

19. Find the definite integral.

$\int_0^{\pi/5} x\cos 5x\,dx$

$u = x$ $dv = \cos 5x\,dx$

$du = dx$ $v = \frac{\sin 5x}{5}$

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$\frac{x\sin 5x}{5} - \int \frac{\sin 5x}{5}dx = \left[\frac{x\sin 5x}{5} + \frac{\cos 5x}{5^2}\right]_0^{\pi/5}$

$\frac{\pi\sin\pi}{25} + \frac{\cos\pi}{25} - \left(\frac{0\sin 0}{5} + \frac{\cos 0}{25}\right) = -\frac{1}{25} - \frac{1}{25} = -\frac{2}{25}$

20. Find the definite integral.

$\int_1^3 x^3 \ln x\,dx$

$u = \ln x$ $dv = x^3dx$

$du = \frac{dx}{x}$ $v = \frac{x^4}{4}$

$\frac{x^4}{4}\ln x - \int \frac{x^4}{4}\frac{dx}{x} = \frac{x^4}{4}\ln x - \frac{1}{4}\int x^3dx$

$= \left[\frac{x^4}{4}\left(\ln x - \frac{1}{4}\right)\right]_1^3 = \frac{81}{4}\left(\ln 3 - \frac{1}{4}\right) - \left(-\frac{1}{16}\right)$

$= \frac{81}{4}\ln 3 - \frac{20}{4} = \frac{81\ln 3 - 20}{4}$

21. Find the indefinite integral.

$\int \cos x \sin^4 x\,dx$

$u = \sin x$

$du = \cos x\,dx$

$= \int u^4 du = \frac{u^5}{5} + C = \frac{\sin^5 x}{5} + C$

22. Find the indefinite integral.

$\int \sin x\cos^3 x\,dx$

$u = \cos x$

$du = -\sin x\,dx$

$= -\int u^3du = \frac{-u^4}{4} + C = -\frac{\cos^4 x}{4} + C$

23. Find the indefinite integral.

$\int \sin^3 4x\cos^4 4x\,dx$

$= \int (1-\cos^2 4x)\cos^4 4x\sin 4x\,dx$

$u = \cos 4x$

$du = -4\sin 4x\,dx$

$-\frac{1}{4}\int (1-u^2)u^4du = \frac{1}{4}\int u^6 - u^4du$

$= \frac{1}{4}\left(\frac{u^7}{7} - \frac{u^5}{5}\right) + C = \frac{1}{4}\left(\frac{\cos^7 4x}{7} - \frac{\cos^5 4x}{5}\right) + C$

24. **Find the indefinite integral.**

$\int \cos^3 4x \sin^2 4x\,dx$

$= \int (1-\sin^2 4x)\sin^2 4x \cos 4x\,dx \quad \frac{1}{4}\int u^2 - u^4\,du$

$u = \sin 4x \quad du = 4\cos 4x\,dx$

$= \frac{1}{4}\left(\frac{u^3}{3} - \frac{u^5}{5}\right) + C$

$= \frac{1}{4}\left(\frac{\sin^3 4x}{3} - \frac{\sin^5 4x}{5}\right) + C$

25. **Find the indefinite integral.**

$\int \cos^3 2x\,dx$

$\int (1-\sin^2 2x)\cos 2x\,dx \quad \frac{1}{2}\int (1-u^2)\,du$

$u = \sin 2x \quad du = 2\cos 2x\,dx$

$= \frac{1}{2}\left(u - \frac{u^3}{3}\right) = \frac{1}{2}\left(\sin 2x - \frac{\sin^3 2x}{3}\right) + C$

26. **Find the indefinite integral.**

$\int \cos^4 5x\,dx$

$\int \left(\frac{1+\cos 10x}{2}\right)^2 dx \quad \frac{1}{4}\int 1 + 2\cos 10x + \cos^2 10x\,dx$

$\int \frac{1+\cos 20x}{2}\,dx$

$\frac{1}{4}\left(x + \frac{2\sin 10x}{10}\right) + \frac{1}{4}\int \cos^2 10x\,dx \quad \frac{1}{2}\left(x + \frac{\sin 20x}{20}\right)$

$= \frac{1}{4}\left(\frac{3x}{2} + \frac{\sin 10x}{5} + \frac{\sin 20x}{40}\right) + C$

27. **Find the indefinite integral.**

$\int \sin^3 \frac{x}{5}\,dx$

$\int \left(1 - \cos^2\left(\frac{x}{5}\right)\right)\sin\frac{x}{5}\,dx \quad -5\int (1-u^2)\,du$

$u = \cos\left(\frac{x}{5}\right)$

$du = -\frac{1}{5}\sin\left(\frac{x}{5}\right)dx$

$-5\left(u - \frac{u^3}{3}\right) = -5\left(\cos\left(\frac{x}{5}\right) - \frac{\cos^3\left(\frac{x}{5}\right)}{3}\right) + C$

$= \frac{5}{3}\cos^3\left(\frac{x}{5}\right) - 5\cos\left(\frac{x}{5}\right) + C$

28. **Find the indefinite integral.**

$\int \sin^2 2x\,dx$

$\frac{1}{2}\int (1-\cos 4x)\,dx = \frac{1}{2}\left(x - \frac{1}{4}\sin 4x\right) + C = \frac{4x - \sin 4x}{8} + C$

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29. **Find the indefinite integral.**

$\int \sin^3 2\theta \sqrt{\cos 2\theta}\,d\theta$

$\int (1 - \cos^2 2\theta)\sin 2\theta \sqrt{\cos 2\theta}\,d\theta = -\frac{1}{2}\int (1-u^2)\sqrt{u}\,du$

$u = \cos 2\theta$

$du = -2\sin 2\theta\,d\theta$

$= \frac{1}{2}\int u^{\frac{5}{2}} - u^{\frac{1}{2}}\,du$

$= \frac{1}{2}\left(\frac{u^{7/2}}{\frac{7}{2}} - \frac{u^{3/2}}{\frac{3}{2}}\right) =$

$\frac{u^{\frac{7}{2}}}{7} - \frac{u^{\frac{3}{2}}}{3}$

$\left(\frac{\cos^3 2\theta}{7} - \frac{\cos 2\theta}{3}\right)\sqrt{\cos 2\theta} + C$

30. Find the indefinite integral.

$\int \frac{\cos^3\theta}{\sqrt{\sin\theta}}\,d\theta$

$\int \frac{1-\sin^2\theta}{\sqrt{\sin\theta}}\cos\theta\,d\theta$

$u=\sin\theta$
$du=\cos\theta\,d\theta$

$\int \frac{1-u^2}{\sqrt{u}}\,du$

$\int u^{-\frac{1}{2}} - u^{\frac{3}{2}}\,du$

$\frac{u^{\frac{1}{2}}}{\frac{1}{2}} - \frac{u^{\frac{5}{2}}}{\frac{5}{2}} + c$

$2\sqrt{\sin\theta} - \frac{2}{5}\sin^2\theta\sqrt{\sin\theta} + c$

31. Find the indefinite integral.

$\int \tan^3 7x\,dx$

$\int (\sec^2 7x - 1)\tan 7x\,dx = \frac{1}{7}\int u\,du - \int \tan 7x\,dx$

$u=\tan 7x$
$du = 7\sec^2 7x\,dx$

$= \frac{u^2}{14} + \frac{1}{7}\ln|\cos 7x| = \frac{1}{14}\left(\tan^2 7x + 2\ln|\cos 7x|\right) + c$

32. Find the indefinite integral.

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$\int \tan^5\left(\frac{x}{11}\right)dx = 11\int \tan^5 z\,dz$

$z=\frac{x}{11}$
$dz=\frac{dx}{11}$

$11\int (\sec^2 z - 1)^2 \tan z\,dz$

$11\int (\sec^4 z - 2\sec^2 z + 1)\tan z\,dz$

$11\left(\int (\sec^3 z - 2\sec z)\sec z\tan z\,dz + \int \tan z\right)$

$u=\sec z$, $du=\sec z\tan z$

$11\int u^3 - 2u\,du + \int \tan z\,dz$

$11\left(\frac{u^4}{4} - u^2\right) + \ln|\sec z| + c$

$\frac{11\sec^4\left(\frac{x}{11}\right)}{4} - 11\sec^2\left(\frac{x}{11}\right) + \ln\left|\sec\left(\frac{x}{11}\right)\right| + c$

33. Find the indefinite integral.

$\int \sec^4 2x\,dx$

$\int (\tan^2 2x + 1)\sec^2 2x\,dx$

$u=\tan 2x$ $du = 2\sec^2 2x\,dx$

$\frac{1}{2}\int (u^2+1)\,du = \frac{1}{2}\left(\frac{u^3}{3}+u\right)+c$

$= \frac{\tan^3 2x}{6} + \frac{\tan 2x}{2} + c$

34. Find the indefinite integral.

$\int \sec^6 4x\,dx$

$\int (\tan^2 4x + 1)^2 \sec^2 4x\,dx = \frac{1}{4}\int (u^2+1)^2\,du = \frac{1}{4}\int (u^4 + 2u^2 + 1)\,du$

$u=\tan 4x$
$du = 4\sec^2 4x\,dx$

$= \frac{1}{4}\left(\frac{u^5}{5} + \frac{2u^3}{3} + u\right) = \frac{1}{4}\left(\frac{\tan^5 4x}{5} + \frac{2\tan^3 4x}{3} + \tan 4x\right) + c$

35. Find the indefinite integral.

$\int \tan^3\left(\frac{\pi x}{7}\right)\sec^2\left(\frac{\pi x}{7}\right)dx$

$u=\tan\left(\frac{\pi x}{7}\right)$

$du = \frac{\pi}{7}\sec^2\frac{\pi x}{7}\,dx$

$\frac{7}{\pi}\int u^3\,du = \frac{7}{\pi}\frac{u^4}{4} + c$

$\frac{7}{4\pi}\tan^4\left(\frac{\pi x}{7}\right) + c$

36. Find the indefinite integral.
$\int \sec^5 3x \tan 3x\, dx$

$$\frac{1}{3}\int u^4\,du = \frac{u^5}{15}+C = \frac{\sec^5 3x}{15}+C$$

$u = \sec 3x$
$du = 3\sec 3x \tan 3x\, dx$

37. Find the arc length of the graph of the function $y=\frac{2}{3}x^{\frac{3}{2}}+2$ over the interval [0,12].

$y' = x^{\frac{1}{2}}$

$$S = \int_0^{12}\sqrt{1+(x^{\frac{1}{2}})^2}\,dx \to \int_0^{12}\sqrt{1+x}\,dx = \int_{x=0}^{x=12} u^{\frac{1}{2}}\,du = \left[\frac{2}{3}u^{\frac{3}{2}}\right]_1^{13} = \frac{2}{3}\left(13^{\frac{3}{2}}-1\right)$$

$u = x+1$
$du = dx$

38. Find the arc length of the graph of the function $y=18x^{\frac{3}{2}}+6$ over the interval [0,7].

$y' = 27x^{\frac{1}{2}}$

$$S = \int_0^7 \sqrt{1+27^2x}\,dx$$

$u = 1+27^2x$
$du = 27^2\,dx$

$$\frac{1}{27^2}\int_{x=0}^{x=7} u^{\frac{1}{2}}\,du = \left[\frac{2}{3\cdot 27^2}u^{\frac{3}{2}}\right]_1^{1+7\cdot 27^2}$$

$$\frac{2}{3\cdot 27^2}\left(\left(1+7\cdot 27^2\right)^{\frac{3}{2}}-1\right)$$

39. Find the arc length of the graph of the function $y=\frac{3}{2}x^{\frac{2}{3}}+10$ over the interval [1,512].

$y' = x^{-\frac{1}{3}}$

$$S = \int_1^{512}\sqrt{1+x^{-\frac{2}{3}}}\,dx \to \int_1^{512} x^{-\frac{1}{3}}\sqrt{x^{\frac{2}{3}}+1}\,du \to \frac{3}{2}\int_{x=1}^{x=512}\sqrt{u}\,du = \left[u^{\frac{3}{2}}\right]_{x=1}^{x=512} = 65^{\frac{3}{2}}-2^{\frac{3}{2}}$$

$u = x^{\frac{2}{3}}+1$
$du = \frac{2}{3}x^{-\frac{1}{3}}dx$

$u=65$, $x=512$
$x=1$, $u=2$

40. Find the arc length of the graph of the function $y=\frac{x^8}{16}+\frac{1}{12x^6}$ over the interval [1,2].

$$y' = \frac{x^7}{2} - \frac{1}{2x^7}$$

$$y'^2 = \frac{x^{14}}{4} - \frac{1}{2} + \frac{1}{4x^{14}}$$

$$S = \int_1^2 \sqrt{1+\frac{x^{14}}{4}-\frac{1}{2}+\frac{1}{4x^{14}}}\,dx = \int_1^2 \frac{x^7}{2}+\frac{1}{2x^7}\,dx$$

$$= \left[\frac{x^8}{16}-\frac{1}{12x^6}\right]_1^2 = \frac{256}{16}-\frac{1}{12\cdot 64}-\left(\frac{1}{16}-\frac{1}{12}\right)$$

$$= 16+\frac{1}{12}-\frac{1}{16}-\frac{1}{12\cdot 64}$$

41. Find the arc length of the graph of the function $x=\frac{1}{3}(y^2+2)^{\frac{3}{2}}$ over the interval $0 \le y \le 14$.

$$\frac{dx}{dy} = \frac{1}{2}(y^2+2)^{\frac{1}{2}}\cdot 2y$$

$$\left(\frac{dx}{dy}\right)^2 = (y^2+2)y^2 = y^4+2y^2$$

$$S = \int_0^{14}\sqrt{1+2y^2+y^4}\,dy \to \int_0^{14}(y^2+1)\,dy = \left[\frac{y^3}{3}+y\right]_0^{14}$$

$$\boxed{\frac{14^3}{3}+14}$$

42. Find the arc length of the graph of the function $x = \frac{1}{3}(y-3)\sqrt{y}$ over the interval $1 \le y \le 9$.

$$\frac{dx}{dy} = \frac{1}{2}y^{\frac{1}{2}} - \frac{1}{2}y^{-\frac{1}{2}}$$

$$\left(\frac{dx}{dy}\right)^2 = \frac{y}{4} - \frac{1}{2} + \frac{y^{-1}}{4}$$

$$S = \int_1^9 \sqrt{1 + \frac{y}{4} - \frac{1}{2} + \frac{y^{-1}}{4}}\, dy$$

$$= \int_1^9 \sqrt{\frac{y}{4} + \frac{1}{2} + \frac{y^{-1}}{4}}\, dy = \int_1^9 \frac{y^{\frac{1}{2}}}{2} + \frac{1}{2}y^{-\frac{1}{2}}\, dy$$

$$\left[\frac{1}{2}\left(\frac{y^{\frac{3}{2}}}{\frac{3}{2}} + \frac{y^{\frac{1}{2}}}{\frac{1}{2}}\right)\right]_1^9 = \left[\frac{y^{\frac{3}{2}}}{3} + y^{\frac{1}{2}}\right]_1^9 = \left(\frac{27}{3}+3\right) - \left(\frac{1}{3}+1\right)$$

$$= 12 - 1 - \frac{1}{3} = 10 + \frac{2}{3}$$

1. Write the definite integral for the area of the surface generated by revolving the curve about the x-axis.

$y=\frac{1}{5}x^3, \quad 0\le x\le 5$

$y' = \frac{3}{5}x^2$

radius = $y = \frac{1}{5}x^3$

$A = 2\pi\int_0^5 \frac{x^3}{5}\sqrt{1+\left(\frac{3}{5}x^2\right)^2}\,dx$

$= \frac{2\pi}{25}\int x^3\sqrt{25+9x^4}\,dx$

2. Write the definite integral for the area of the surface generated by revolving the curve about the x-axis.

$y=5\sqrt{x}, \quad 4\le x\le 5$

$y' = \frac{5}{2\sqrt{x}}$

radius = $y = 5\sqrt{x}$

$A = 2\pi\int_4^5 5\sqrt{x}\sqrt{1+\left(\frac{5}{2\sqrt{x}}\right)^2}\,dx$

$= 10\pi\int_4^5 \sqrt{x+\frac{25}{4}}\,dx$

3. Write the definite integral for the area of the surface generated by revolving the curve about the y-axis.

$y=\sqrt[3]{x}+11, \quad 1\le x\le 1331$

$(y-11)^3 = x$

$\frac{dx}{dy} = 3(y-11)^2$

radius $= x = (y-11)^3$

$A = 2\pi\int_{x=1,\ y=12}^{x=1331,\ y=22} (y-11)^3\sqrt{1+\left(3(y-11)^2\right)^2}\,dy$

$A = 2\pi\int_{u=1}^{11} u^3\sqrt{1+9u^4}\,du$

4. Write the definite integral for the area of the surface generated by revolving the curve about the y-axis.

$y=100-x^2, \quad 0\le x\le 10$

$y' = -2x$

radius = x

$A = 2\pi\int_0^{10} x\sqrt{1+(-2x)^2}\,dx$

$= 2\pi\int_0^{10} x\sqrt{4x^2+1}\,dx$

5. Write the definite integral for the area of the surface generated by revolving the curve about the y-axis.

$y=\sqrt{9-x^2}, \quad 0\le x\le 2$

$x^2+y^2=9$

$2x\,dx+2y\,dy=0$

$\Rightarrow \frac{dy}{dx} = -\frac{x}{y}$

radius=6

Area $= 2\pi\int_0^2 x\sqrt{1+\frac{x^2}{y^2}}\,dx$

$= 2\pi\int_0^2 x\sqrt{\frac{x^2+y^2}{y^2}}\,dx = 6\pi\int_0^2 \frac{x}{\sqrt{9-x^2}}\,dx$